PSD Analysis for Determining Delay Spread

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Abstract

This paper describes how to determine the expectation and standard deviation of an arbitrary power spectral density (PSD). This is done in the context of propagation where the PSD corresponds to the power delay profile (PDP) and the standard deviation corresponds to the average RMS delay spread.

Continuous, integral based equations are presented and a MATLAB implementation is shown to see how a discrete implementation can be implemented.

1 Introduction

In this document, total power, mean, and standard deviation is calculated. This corresponds to total power, average mean delay, and average RMS delay spread in the context of delay spread. The notation is as follows:

- $P(t)$ = Power spectral density = power delay profile (PDP(t)).
- P_t = Total power of the PSD/PDP.
- μ = Expectation = Average mean delay.
- σ = Standard deviation = average RMS delay spread.

The continuous-time equations will be described below. In the end, a MATLAB implementation will be shown, to show a discrete implementation.

2 Total Power

If the PSD is normalized so it has an area of 1, then the PSD becomes an approximation of the PDF of the process. Therefore, it is relevant to determine the total power of the PSD to use it for normalizing later on.

$$P_t = \int_{-\infty}^{\infty} P(t)\,dt \tag{1}$$

3 Expectation—Average Mean Delay

The expectation of a PSD corresponds to the average mean delay of a PDP.

$$\mu = \frac{\int_{-\infty}^{\infty} tP(t)\,dt}{P_t} \tag{2}$$

4 Standard Deviation—Average RMS Delay Spread

The standard deviation of a PSD corresponds to the average RMS delay spread of a PDP.

$$\sigma = \sqrt{\frac{\int_{-\infty}^{\infty} t^2 P(t)\,dt}{P_t} - \mu^2} \tag{3}$$

5 MATLAB Implementation

In Listing 1, a MATLAB implementation of the above equations. The `trapz(x,y)` function performs a numerical integration of `y` along the axis `x`.

```
%% PSD Analyze
% Input:   x = x axis (e.g. time or frequency)
%          P = Arbitrary Power Spectral Density
% Output: Pt = Total power in the PSD
%          mu = Average mean delay/Expectation
%          sig = Average RMS delay spread
%                /Standard deviation
% Example:
%     A = 1;
%     a = 986.82e3;
%     x = 0:100e-12:15e-6;
%     P = A*exp(-a*x);
%     [Pt,mu,sig] = psdanalyze(x,P)
function [Pt,mu,sig] = psdanalyze(x,P)
    Pt = trapz(x,P);
    mu = trapz(x,x .* P) / Pt;
    sig = sqrt( trapz(x, x.^2 .* P)/Pt - mu^2);
end
```

Listing 1: MATLAB implementation that determines total power, expectation, and standard deviation of a PSD.

5.1 Example

As an example of the above MATLAB implementation, a discrete Gaussian PDF, $P \sim \mathcal{N}(20, 16)$ will be shown. The output of `psdanalyze()` is of course a total power of 1, a mean of 20, and a standard deviation of $\sqrt{16} = 4$.

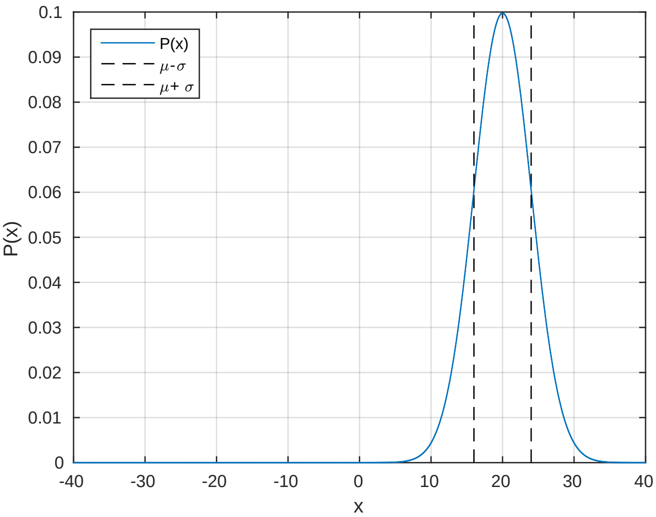

Figure 1: Gaussian PDF and its standard deviation.

The PDF of this Gaussian is

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (4)$$

$$= \frac{1}{4\sqrt{2\pi}} e^{-\frac{(x-20)^2}{2\cdot 4^2}} \quad (5)$$

The example code is shown in Listing 2. The PDF, along with the standard deviation, is shown in Figure 1.

```
%% Example of using psdanalyze().
m = 20;
s = sqrt(16);
x = linspace(-40,40,1000);
P = 1/(s*sqrt(2*pi)) * exp(-(x-m).^2./(2*s^2));
[Pt,mu,sig] = psdanalyze(x,P)
plot(x,P); hold on
plot([mu-sig,mu-sig], [0, 0.1], '--k');
plot([mu+sig,mu+sig], [0, 0.1], '--k'); hold off
legend('P(x)', '\mu-\sigma', '\mu+\sigma', 'location',
     'northwest')
xlabel('x'); ylabel('P(x)'); grid on
```

Listing 2: Example of using `psdanalyze()` on a Gaussian PDF.

The output of the example in Listing 2 is

```
Pt =
      1
mu =
     20
sig =
      4
```

as expected.