Solving homogenous linear differential equations

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This method is used to find the solutions to an equation og the form

$$ay'' + by' + cy = 0$$

To solve this we rewrite the equation to try to guess a root (some roots):

$$ar^2 + br + c = 0$$

Now we find the roots for this equation. First the discriminant, to determine the method for finding roots:

$$D = b^2 - 4ac$$

Now the solutions depend on D:

1 For $D > 0$

$$r = \frac{-b \pm \sqrt{D}}{2a}$$

Now the solution for the initial equation is:

$$\boxed{y(t) = c_1 \exp(r_1 t) + c_2 \exp(r_2 t)} \quad (1)$$

2 For $D = 0$

$$r = \frac{-b}{2a}$$

$$\boxed{y(t) = c_1 \exp(rt) + c_2 t \exp(rt)} \quad (2)$$

3 For $D < 0$

This gives a complex solution.

Rewrite the sqare root including i:

$$\sqrt{|D|}i$$

and solve the complex roots:

$$r = \frac{-b \pm \sqrt{|D|}i}{2a}$$

yields conjunctive roots, $\alpha \pm \beta i$.

Now the solutions are given with:

$$y(t) = c_1 \exp(\alpha t) \cos(\beta t) + c_2 \exp(\alpha t) \sin(\beta t) \quad (3)$$