# Simplifying trigonometric products into trigonometric sums

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It can be hard to differentiate products of trigonometric functions like this one:

$$\cos y \sin y$$

It is easier when the functions are separated like this:

$$\cos y + \sin y$$

for example. Therefore I will here describe how to transform equations of the first form into equations of the second form using the Euler's formula.

Euler's formula lets us transform the sin and cos parts into this:

$$\cos(y) = \frac{e^{iy} + e^{-iy}}{2}$$
$$\sin(y) = \frac{e^{iy} - e^{-iy}}{2i}$$

By doing this we can make use of the rules for products of e, which are preserved for complex numbers:

$$e^{z_1} \; e^{z_2} = e^{z_1 + z_2}$$

and magically doing this will end up with some parts that can be put back on the regular trigonometric form be reversing Euler's formula.

## 1 Worked example

I will now rewrite trigonometric products like this:

$$\sin 3x \cos 5x = \frac{1}{2}(\sin 8x - \sin 2x).$$

First, I will rewrite the left side using Euler's formula:

$$\left(\frac{e^{3xi} - e^{-3xi}}{2i}\right)\left(\frac{e^{5xi} + e^{-5xi}}{2}\right).$$

I pull the denominator out in front, and use the product rules of e:

$$\frac{1}{4i}(e^{3xi+5xi} + e^{-3xi+5xi} - e^{-3xi+5xi} - e^{-3xi-5xi})$$
$$= \frac{1}{4i}(e^{8xi} + e^{-2xi} - e^{2xi} - e^{-8xi})$$

I break it up in two parts:

$$\frac{1}{4i}(e^{8xi} - e^{-8xi}) + \frac{1}{4i}(e^{-2xi} - e^{2xi})$$

Now what we are trying to do is to get it on the form defined by Euler's formula. Because there is a minus between the e-s in both the first and the second part, we assume that we want to go back to sinus in both cases. Sinus was defined as:

$$\sin(y) = \frac{e^{iy} - e^{-iy}}{2i}$$

and therefore part of the denominator of $\frac{1}{4i}$, is "pulled" into the equation like this:

$$\frac{1}{2}\left(\frac{e^{8xi} - e^{-8xi}}{2i}\right) + \frac{1}{2}\left(\frac{e^{-2xi} - e^{2xi}}{2i}\right)$$

Now the "equation" is on the Euler's form, and can be put back as an ordinary trigonometric function. Taking the signing into condition, it ends up like this:

$$\frac{1}{2}(\sin 8x - \sin 2x)$$

and thereby the first statement is proved.